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A STUDY ON THE VIBRATION CHARACTERISTICS OF A LARGE SIZE KOREAN BELL

J. M. Lee

School of Mechanical and Aerospace Engineering, Seoul National University, San 56-1, Shillim-dong, Kwanak-gu, Seoul 151-742, Korea. E-mail: leejm@gong.snu.ac.kr

S. H. Kim

Department of Precision Mechanical Engineering, Kangwon National University, Chuncheon, Kangwon Do, Korea

S. J. Lee

Samsung Electronics Co., Ltd., Suwon, Kyungi Do, Korea

J. D. Jeong

School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea

H. G. Choi

Turbo and Power Machinery Research Center, Seoul National University, Seoul, Korea

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Structural elements which determine the vibration and sound properties of the Korean bell are reviewed. Vibration response characteristics under an impact excitation are investigated by the analysis and experiment. Numerical and experimental methods to predict and tune the vibration and sound properties of a large Korean bell (named the New Bosingak Bell) are introduced. Beat phenomenon, which is a very important sound property of the Korean bell, is analytically examined, and an experimental technique to enhance the beat property is proposed.

1. INTRODUCTION

Korean bells, manufactured since the ancient Silla dynasty (57 B.C.–A.D. 935), are unique in shape and have beautiful sounds and appearances. Although the forms of the ancient Korean bells have varied with the eras, the profile, like a barrel, and the beautiful engravings on the body are the typical characteristics of the Korean bell. The large Korean bell (more than 20 tons) has the breathing-like beat of the hum tone, about 60–65 Hz with 3–5 s beat period, and it is regarded having a beautiful sound. There have been studies to scientifically uncover the secret of the beauty of the ancient Korean bells [1–3], to understand the essence of the sound of Korean bells and to aid in the design and casting of modern bells. The sonority of a bell is determined by the relationships among frequencies, intensities, and the decay times of partials. At the instant of impact, most dominant sound qualities are produced by several partial tones. However, higher mode tones decay rapidly after striking and people hear just the sound produced by the first (hum tone) and the second mode (fundamental tone), which lasts long.

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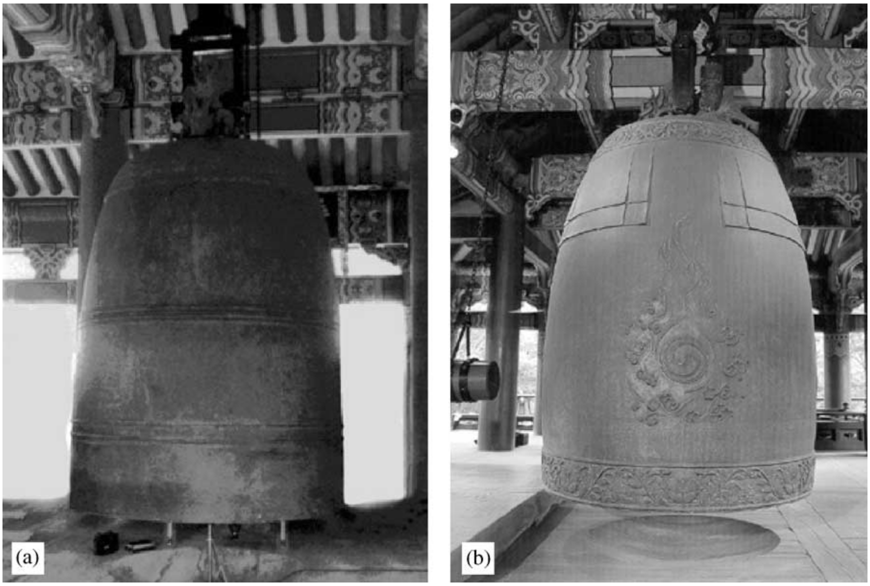

Figure 1. Bosingak bell: (a) old bell (b) new bell.

The natural frequencies of bell vibration modes are basically determined by the main dimensions of the nearly axisymmetric body and the material properties of the bell. However, sculptures, carved figures on the body and casting irregularities cause slight asymmetry, generating beating sounds which are deemed to be beautiful and desirable in Korean bells. People feel the depth and the liveliness of a bell sound by this beating sound, and a clear beat with a proper period is one of critical factors in the appreciation of the Korean bell sound. On the other hand, the harmony of partials is a main factor in the Western bell and the phenomenon of warble is often considered as a flaw, which needs to be kept within reasonable limits [4–6].

Once the dynamic properties of the bell are known, the striking position becomes another critical factor for a good beat. The striking position is designed to be located at the center of percussion in height and simultaneously at the proper position on the circumference for a clear beat. Another important factor in the sound of the Korean bell is the sound duration time, which is directly influenced by modal damping determined by chemical composition, cooling speed of casting and so on. Usually, a long lasting hum or fundamental tone is desired. Supplementary acoustic elements, such as a sound pipe on top of the bell and a sound hole below the bell, also influence the sound quality.

This paper introduces how the above design points are applied to manufacturing a large Korean bell, named the New Bosingak Bell (Figure 1(b)). The bell was manufactured in 1987 to replace the Old Bosingak Bell (Figure 1(a)), which was the second largest bell in Korea, cast in 1468. In the design stage of the New Bosingak Bell, characteristics of all the important low-frequency modes are predicted numerically and impact response characteristics are examined. In particular, a post-casting experimental technique for producing a clear beat and a proper beat period for a large Korean bell is proposed.

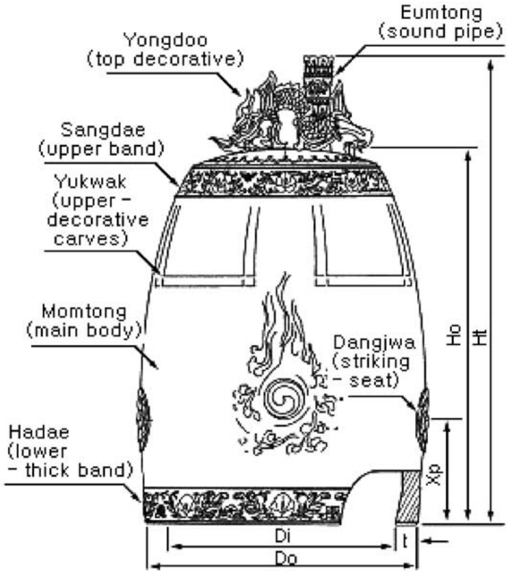

Figure 2. Design variables of a typical Korean bell.

2. TYPICAL KOREAN BELL

As in other typical Korean bells, the New Bosingak Bell is structurally composed of *Momtong* (main body), *Hadae* (lower thick band), *Sangdae* (upper band), *Dangjwa* (striking seat), *Bichun* (lower decorative), *Yukwak* (upper decorative carves), *Yongdoo* (top decorative) and *Eumtong* (sound pipe). The material is composed of Cu (83·7%), Sn (16·2%) and a small amount of Fe, Zn, Pb and Ni. Figure 2 shows some important design variables of the Korean bell. The primary low frequency modes are determined by the dimensions of these principal variables, and can be predicted numerically.

The symmetry of the main body is broken by decorative ornaments, sculptures on the surface and by metallurgical imperfection during the cast. Although this slight asymmetry has no significant effects on the natural frequencies of lower modes, it causes the circumferential mode pairs to split and produces the beat phenomenon.

The New Bosingak Bell, which was designed and supervised up to its casting by the Engineering Faculty of the Seoul National University, is shown in Figure 1. It weighs about 20 tons and its main dimensions and material properties are shown in Table 1.

3. ANALYSIS AND DESIGN OF BELL STRUCTURE VIBRATIONS

3.1. PRELIMINARY DESIGN

The preliminary design of a bell was made, based on the database of other ancient Korean bells. Generally it is hard to mathematically represent the structural vibrations of a bell, so numerical or empirical methods are used to predict the vibration characteristics. In the design stage, a simplified mathematical model with general dimensions and material properties may be sufficient to estimate the natural modes in the low-frequency range. First, the bell is simplified as a cylindrical shell and the basic geometries are determined with appropriate approximation. The natural frequency and corresponding radial displacement

Table 1

Main dimensions and material properties of New Bosingak Bell [( ): *dimensions of the old model*]

Design variables (see Figure 2)	Main dimensions (mm)
Outer diameter (D_o)	2224 (2228)
Inner diameter (D_i)	1836 (1588)
Height of main body (H_o)	3080 (2880)
Total height (H_t)	3825 (3647)
Height of *Dangjwa* (X_p)	847 (600)
Thickness of *Hadae* (t)	194 (320)
Material properties	Young's modulus = 91 GPa Mass density = 8800 kg/m^3 The Poisson ratio = 0·34

mode of a cylindrical shell can be estimated from the well-known in-extensional approximation, as follows [7]. Here, a is the radius, L is the height, h is the thickness of a cylindrical shell, θ is the angular co-ordinate and ϕ is an arbitrary phase shift. And (m, n) is a mode number in axial and circumferential directions.

$$\omega_{mn} = \sqrt{\frac{E}{12\rho(1-\mu^2)}}\left(\frac{h}{a}\right)\frac{1}{a}\left[\left(m\pi\frac{a}{L}\right)^2 + n^2\right], \tag{1}$$

$$u_{mn} = A\sin\left(\frac{m\pi x}{L}\right)\cos n(\theta-\phi)\mathrm{e}^{\mathrm{j}\omega_{mn}t} \tag{2}$$

where E, ρ and μ are Young's modulus, mass density and the Poisson ratio of the material respectively.

For more accurate estimation of vibration characteristics, a ring-stiffened cylindrical shell with an attached concentrated mass can be used as a simplified model [8]. From this model, the beat characteristics of the Korean bell can be predicted and analytically examined. By altering the geometry and stiffening effect by the lower thick band (*Hadae*), a bell designer will have sufficient alternatives to control the natural frequencies before the cast. The preliminary design is followed by the finite element analysis (FEA) and more accurate predictions of natural frequencies and mode shapes are made. Design modification and analysis are repeated to obtain the proper modal data. Another important design variable to be determined in designing a bell is the position of the striking seat (*Dangjwa*). Using the theory of a center of percussion, the distance of the striking position from the hinged point in Figure 3 is determined as follows [9]:

$$q = R_G + \frac{I_G}{MR_G}, \tag{3}$$

where R_G is the distance from the hanger to the mass center of the bell, M the total mass of the bell and I_G the mass moment of inertia about the mass center.

When this sweet spot (the center of percussion) is struck, the reaction force at the hinged position is minimized. This both lengthens the life of the bell hanger and makes

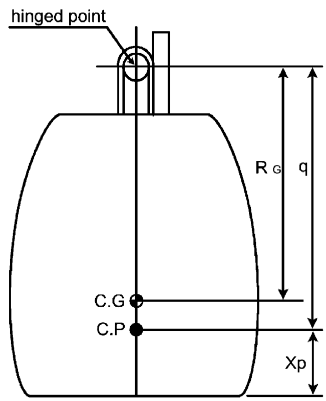

Figure 3. A center of percussion in the bell structure.

a long lingering sound by minimizing the friction on the hanger. The center of percussion of the New Bosingak Bell was calculated as 847 mm above the lower end of the bell by FEA.

3.2. IMPULSE RESPONSE ANALYSIS AND BEAT CHARACTERISTICS

Asymmetry of a bell produces a beat phenomenon for each circumferential mode. Beat characteristics in the impulse response can be understood using the theory of a cylindrical shell with slight asymmetry. For a slightly asymmetric cylindrical shell, a radial component of vibration displacement is given as a pair of lower (L) and higher (H) modes with the same circumferential mode number n [7].

$$U_{mn,L}(x,\,\theta) = \Phi_{\mathrm{m}}(x)\cos n\theta, \tag{4}$$

$$U_{mn,H}(x,\,\theta) = \Phi_m(x)\cos n\left(\theta - \frac{\pi}{2n}\right) \tag{5}$$

where $\Phi_m(x)$ and $\cos n\theta$ are the axial and circumferential modal functions respectively.

Since the amount of asymmetry is slight, the mode pair is considered to have same mode shape with a phase difference, $\pi/2n$, and the natural frequencies of the mode pair are very close to each other. When an impulse load is applied to a bell, the mode pair is excited simultaneously. For a cylindrical shell having slight asymmetry, the radial component of vibration velocity response has been investigated as follows [8].

$$\dot{w}(x,\theta,t) = \sum_m \sum_n \frac{F\Phi_m(x^*)\Phi_m(x)}{N_m}\left[\cos n\theta^* \cos n\theta \cos(\Omega_{mn,L}t) + \sin n\,\theta^* \sin n\theta \cos(\Omega_{mn,H}t)\right], \tag{6}$$

where $N_m = \pi a \int_0^L \rho h \Phi_m^2(x)\,\mathrm{d}x$, and a, h and ρ are the radius, thickness and mass density of a shell, respectively, and F denotes the point impulse at position (x^*, θ^*). In equation (6), the slight difference between $\Omega_{mn,L}$ and $\Omega_{mn,H}$ results in a beat phenomenon, whose period is calculated by the difference of the two frequencies.

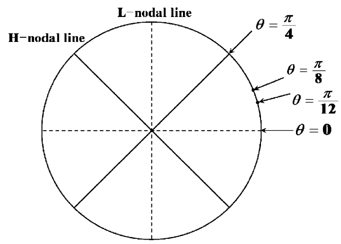

Figure 4. Nodal lines for $n = 2$ and striking, response points (bottom view).

TABLE 2

Natural frequencies of the first and second modes

Method	Mode (n, m)		Frequency (Hz)	Beat (Hz)
Calculation	(2,0)	Low	62·92	0·13
		High	63·05	
	(3, 0)	Low	160·0	0·2
		High	160·2	
	(2, 1)	Low	201·9	0·2
		High	202·1	
	(3, 1)	Low	237·5	0·5
		High	238·0	
Experiment	(2, 0)	Low	62·10	0·27
		High	62·37	
	(3, 0)	Low	162·32	0·15
		High	162·47	
	(2, 1)		200·23	
	(3, 1)		235·02	

As can be seen in equation (6), the clarity of a beat depends on the circumferential striking position and a clear beat is generated when the H and L modes are excited equally. Therefore, the striking position (x^*, θ^*) should be on the mid-point of nodal lines of the H and L modes. To see the distribution of the beat on the surface of the bell, the impulse response is investigated on the first natural mode under different excitations and at different response positions as shown in Figure 4. In the calculation, the natural frequencies of the (2, 0) modes in Table 2 are substituted into equation (6), and the magnitude is normalized so that the maximum velocity amplitude equals one. Figure 5 shows the impulse responses of the $n = 2$ mode when striking the anti-node of the H mode ($\theta^* = 0$). Only the H mode is excited, and vibration responses at all points show no beat, therefore no beat is heard in the sound. Striking at $\theta^* = \pi/8$, however, equally excites the mode pair, and a clear beat is produced at some positions between L and H nodal lines. As shown in Figure 6(c), the clear beat is observed at $\theta = \pi/8$ since the L and H modes equally respond at this point. This phenomenon will be the same at other mid-points of L and H nodal lines, i.e., at $\theta = [2n + 1]\pi/8$ ($n = 0, 1, 2, \dots$). The bell will strongly radiate the clear beat in this striking condition. Consequently, the mid-point of L and H nodal lines of $n = 2$ mode is chosen as the optimal striking position to generate the clear beat and a long lingering sound. However, the problem lies in the fact that the accurate position cannot be predicted because of casting irregularity.

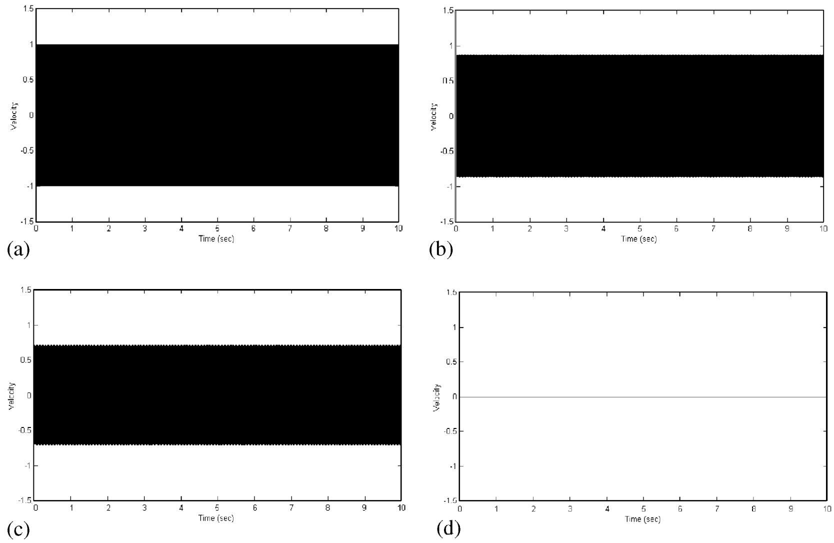

Figure 5. Impulse responses under striking at point $\theta^* = 0$: (a) $\theta = 0$, (b) $\theta = \pi/12$, (c) $\theta = \pi/8$, (d) $\theta = \pi/4$.

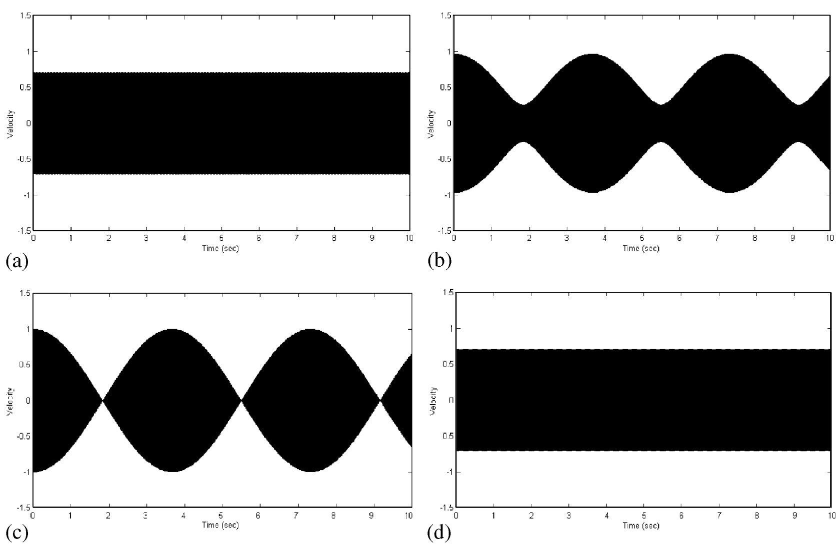

Figure 6. Impulse responses under striking at point $\theta^* = \pi/8$: (a) $\theta = 0$, (b) $\theta = \pi/12$, (c) $\theta = \pi/8$, (d) $\theta = \pi/4$.

3.3. FINITE ELEMENT ANALYSIS

Analysis of vibration characteristics by FEA can afford understanding of the clear beat and clues to control the nodal lines with minor modifications. Commercial finite element analysis software ANSYS [10] is used to predict the natural frequencies and to design sound properties. The New Bosingak Bell is modelled with 2736 elements to calculate its natural frequencies and the effects of the localized thickness irregularities are simulated. As part of this study, an efficient computing algorithm using ring elements was also developed to predict modal data of a slightly asymmetric shell [11] and detailed modelling techniques for bell structures are reported [12]. The natural modes and frequencies are shown in

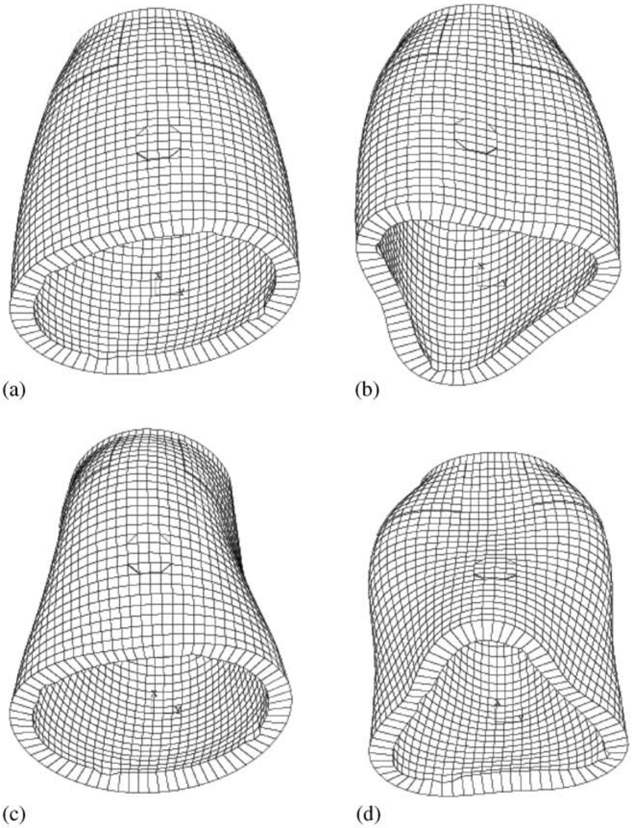

Figure 7. Natural mode shapes of the bell structure: (a) (2, 0) mode, (b) (3, 0) mode, (c) (2, 1) mode, (d) (3, 1) mode.

Figure 7 and Table 2. In this figure, (2, 0) mode means the mode which has mode number of $n = 2$ and $m = 0$.

The phase difference of the doublet is 45° for $n = 2$ mode, and 30° for $n = 3$ mode. In Table 2, calculated and measured natural frequencies are very close, indicating that the finite element modelling is adequate to predict the vibration characteristics in the design stage. However, beat frequencies by FEA do not agree with those obtained by experiment, this is thought to be unavoidable due to the unpredictable asymmetric factors described above.

4. EXPERIMENTAL TECHNIQUE FOR TUNING VIBRATION AND SOUND

4.1. TUNING STRATEGIES

The striking seat (*Dangjwa*) should be at the mid-point of the L and H nodal lines to produce a clear beat. The designed striking seat, however, is not always placed at the center of the L and H nodal lines because of casting irregularities; sometimes, it may even be adjacent to one of the nodal lines. This unpredictable situation can be improved by structural modification after cast, shifting the nodal lines to make *Dangjwa* the proper striking point. Based on the results by numerical simulation, artificial thickness

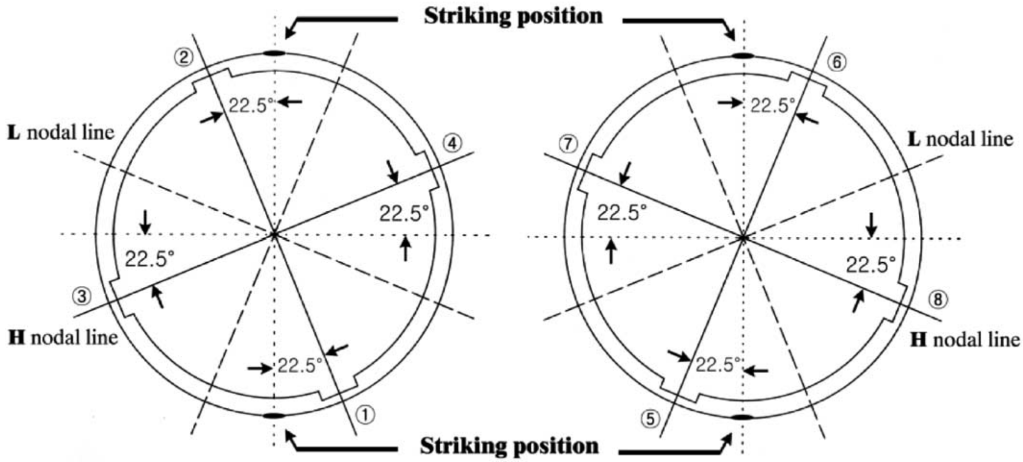

Figure 8. Cut positions for tuning the beat of the first mode.

perturbation is applied to dilute the complicated asymmetry effects. A local small cut in the symmetric shell decreases the bending stiffness around the cut, and the anti-node of the L mode in equation (4) moves toward the cut in proportion to depth of the cut. Based on this theory, the inside of the lower thick band (*Hadae*) is ground step-by-step. This method is also applied to tune the Western bell [12]. Theoretical analysis or FEA is necessary prior to the experimental tuning. In this study, the size and depth of the cut are estimated by the theoretic analysis results. As shown in Figure 8, four positions (①, ② and ⑤, ⑥) or other four equivalent positions (③, ④ and ⑦, ⑧) are the proper cut positions, depending on the distribution of nodal lines. Usually considering the strength reduction by the local cut, the positions far from the striking seat are favored for modification. Cheon and Lee have performed numerical simulation on the effect of various asymmetric elements of a bell structure including a cut [13]. Hong and Lee have proposed the analytical method to predict the modal property of slightly asymmetric ring having a small cut or fault [14].

4.2. PROCEDURE AND RESULTS OF TUNING UP

Prior to modification by grinding, the distribution of nodal lines of the L and H modes is found by modal test. Usually an impact test and resonance test are used. Firstly, the spectral properties of vibration and sound are identified by the impact test and then the position of nodal lines is found by the resonance test using a loudspeaker as an exciter. The resonance test is especially useful to trace the shift of the nodal lines. Natural frequencies of the two low-frequency mode pairs measured by impact test are compared with those calculated by FEA in Table 2. Figure 9(a) shows the configuration of L and H nodal lines of the (2, 0) mode before modification. A nodal line of the L mode (an anti-node of the H mode) passes around the striking seat (*Dangjwa*). Therefore, the L mode is very weakly excited, and the beat of the first mode is not audible. In order to tune up the bell, the positions ⑦ and ⑧ shown in Figure 8 are adopted as the cut area, since these positions are far from the striking seats. For the same perturbation effect and structural safety, distributed grindings at several positions are desirable.

At first, a 30×30 cm area inside the bell is ground by 1 cm depth at the position ⑧, 10 cm above the lower end. Due to the first grinding, the L-nodal line rotates by 7°

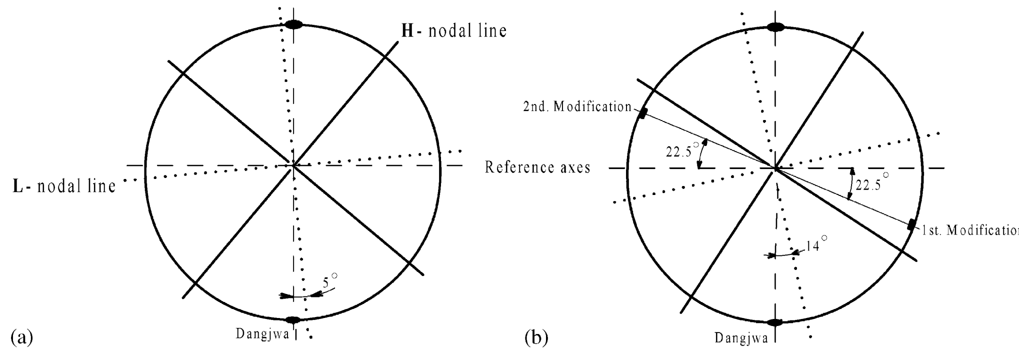

Figure 9. Modifications and resulting nodal lines positions in New Bosingak Bell: (a) before modification (b) after modification

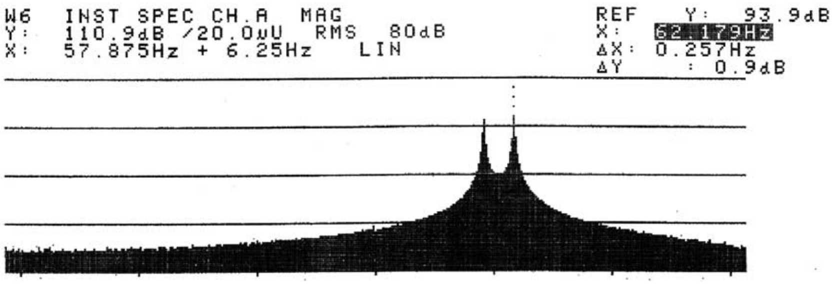

Figure 10. Power spectrums of (2,0) modes (measured at striking seat by accelerometer).

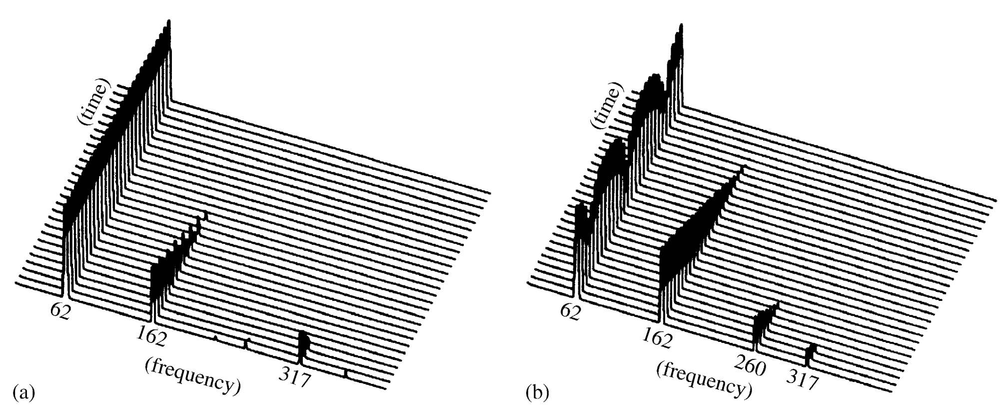

Figure 11. Comparison of vibration spectrum of New Bosingak Bell: (a) before tuning, (b) after tuning.

counterclockwise. Secondly, at the position ⑦, a same size area is ground by the same depth; the L mode nodal line rotates by more 2° counterclockwise. After modification, the L nodal lines of the (2, 0) mode move to new positions, 14° from *Dangjwa* as shown in Figure 9(b) and the beat has about a 4 s period. Generally a clear beat with over 4 s is estimated to be a good sound property for a large Korean bell. More modification can place

Dangjwa at the exact mid-point of the *L* and *H* nodal lines. However, the amounts of grinding cut should be adjusted with consideration of bending strength weakening and the period of the beat. Figure 10 shows the power spectrum of acceleration, measured at the position of *Dangjwa* after final tuning. As shown in this figure, the *L* and *H* modes respond with nearly the same magnitude, although the *H* mode is a little stronger than the *L* mode.

The waterfall plot of the impact response is shown in Figure 11 and it confirms the effect of tune up. No beat was initially observed, but the beat of the (2,0) mode was clear and long lasting after tune up. The same procedure also can be performed for the other higher modes. However tuning for the beat of another mode can spoil the tuned beat of the (2,0) mode, because there is no connection between the locations of the nodal lines. Multiple tuning, considering two or more modes simultaneously, may be possible, but it is not easy to calculate and is time consuming. In this study, considering the importance of the first vibration mode, tuning is applied just to the first (2,0) mode.

5. CONSIDERATIONS ON THE SUPPLEMRNTARY DESIGN ELEMENTS

In addition to the above main design factors, some supplementary devices are considered in the design stage for enhancing the sound quality. Sound pipe (*Eumtong*), sound hole (*Myŏngdong*) and striking hammer (*Dangmok*) are valuable parts influencing the vibration and sound properties of the Korean bells. The sound pipe is located at the top of a bell and acts as a sound filter. Among the sounds generated from the inner surface of the bell, high-frequency components are passed and low tones are blocked by this sound pipe. Yum has experimentally confirmed the filtering effect on the sound quality of Korean bells [15] and Lee has reported resonant transmissibility of the sound pipe for the (3,0) mode in a traditional Korean bell [16]. The sound hole is also designed to enhance the sound properties. It is constructed below the bell so that the hole resonates with the frequency of the first or the second vibration mode. This assists the hum tone or fundamental tone to last longer. Finally, the striking system is carefully designed. The quality of the impact sound is strongly influenced by the striking conditions: size, weight and material of the hammer. Usually, a striking hammer is made of wood to strongly excite the low-frequency bending modes and it produces the magnificent and deep sound desired in large Korean bells.

6. CONCLUSIONS

In this paper, the vibration and sound characteristics of the Korean bell are introduced and some design techniques are proposed and reviewed. Ideal striking position to lengthen the life of a bell hanger and to make long lingering sound is proposed and important low-frequency modal data of the Korean bell are predicted by FEA in the design stage. By the analytic approach, the fact that the clarity of a beat depends on the striking position is demonstrated and variant responses on the surface of a bell are simulated. As an alternative to the design of the proper striking position in the design stage, which is not possible because of other asymmetric factors including casting irregularity, we proposed experimental technique to tune the beat property of a bell after cast. This tuning technique was applied to the manufacture of a large Korean bell, named the New Bosingak Bell and we could get improved clear beat properties with a reasonable beat period. For greater understanding of the Korean Bell, we reviewed some supplementary devices, which are designed to enhance the sound quality.

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